**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: We have a normal distribution with *μ* = 45 minutes and *σ* = 8 minutes.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50

Find out To finish in one hour you must have X ≤ 50

Pr(X > 50) = 1 -Pr(X ≤ 50).Z

= (X - *μ*)/*σ* = (X -45)/8.0

Pr(X ≤ 50) = Pr(Z ≤ (50 -45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will

= 100-73.4 = 0.2676 or 26.6%

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Normally distributed with mean *μ* = 38 and Standard deviation *σ* =6

1. Probabilty of employees greater than age of 44= Pr(X>44)

Pr (X > 44) = 1 – Pr (X ≤ 44).

Z = (X -*μ* )/*σ* = (X - 38)/6

Thus the question can be answered by using the normal table to find

Pr (X ≤ 44) = Pr (Z ≤ (44 - 38)/6) = Pr (Z ≤ 1) =84.1345%

Probability that the employees will greater than44 = 100-84.1345=15.86

Probability of employees between 38&44 =Pr(x<=44)-Pr(x>=38)

Pr(x>=38) = 0.5

Therefore, Pr(x<=44)-Pr(x>=38) = 0.84134-0.5 = 0.34134 (or) 34.134%

Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

1. Probabilty of employees less than age of 30 = Pr(X<30)

Z = (X -*μ* )/*σ* = (30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr (X ≤ 30) = Pr (Z ≤ (30 - 38)/6) = Pr (Z ≤ -1.333) =9.12%

So the number of employees with probability 0.912 of them being under age 30

= 0.0912\*400=36.48 ( or 36 employees).

Therefore the statement B of the question is also TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: Ans- As we know that if X = N(µ1, σ1^2 ), and Y = N(µ2, σ2^2 ) are two independent random variables then

X + Y = N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y = N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e. Z is linear combination of X and Y , then Z = N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore, in the question

2X1 = N(2 µ,4 σ^2) and

X1+X2 = N(µ + µ, σ^2 + σ^2 ) = N(2 µ, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: The probability of getting value between a&b is 0.99

So, the probability of getting value outside a&b is 1-0.99=0.01

The probability towards left of a =-0.01/2=-0.05

The probability towards left of a =0.01/2=0.05

By finding std normal variable(z), need to calculate X:

Z = (x- µ)/σ

For a probability of 0.05, Z value is -2.57

Z\*σ+ µ=x

2.57\*20+100=151.4

-2.57\*20+100=48.6

D is the correct option

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Mean profits of two different divisions of a company = mean1+mean2 = 12

Mean profits in Rs =11\*45 = 540 million

Variance of profits from two different divisions of a company = 25

Standard deviation is Rs 225 million

(a) print(‘range is Rs’,(stats.norm,interval(0.95,540,225)),’in Millions’)

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

(b) X= µ+Z (σ) [ from Z table 5 percentile =-1.645 ]

X= 540+(-1.645) \*(225)

5th percentile of profits Rs 170 million

(c) Probability of division 1 making a loss = 0.0477903522728147

Probability of division 1 making a loss = 0.040059156863817086

Probability of Division 1 making a loss in a given year is more than Division 2.